Continuous measurement of photon number: information and state reduction

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Contents

1 Introduction - Why continuous measurement ?

In this paper, we will discuss in various examples effects of information readout or discarding on state reduction from the viewpoint of continuous measurement of photon number. We would like to give you some idea of what we have been thinking about this matter, and are looking forward to attaining further developments in our studies in light of stimuli we hope to reveive from you.

In general, a quantum measurement process plays two distinct roles with respect to the past and future of the observed system. With respect to the past, it verifies the probability distribution of an observable for an pre-measurement quantum state by repeated measurements, namely, by performing the same measurement *many times*. On the other hand, with rewpect to the future, it produces a post-measurement quantum state via nonunitary state reduction by a *single* measurement. Such an asymmetry with respect to directions of time is the source of irreversibility inherent in a quantum measurement process and distinguishes quantum mechanics from classical mechanics.

According to von Neumann's quantum theory of measurement, a quantum measurement process is catagorized into two stages. In the first stage, a quantum correlation must be established between the system and the measuring apparatus via a unitary interaction. This process is reversible because the interaction is unitary. In the second stage, the apparatus meter is readout instantaneously, causing a nonunitary state reduction of the system. Thus, the measurement process is irreversible only at the second stage.

An actual photodetection process, however, differs essentially from this picture because the number of photoelections is measured not at a single time but one by one. In the following we consider the time evolution of a photon field in a closed optical cavity. The photodetector begins to count photons when a small window on the cavity wall is opened. Information concerning registration of a photocount is read out in real time throughout the measurement period. The state reduction of the photon field therefore occurs at every moment when the detector is active, and the photon field thus evolves nonunitarily.

The nonunitary time development of the photon field is schematically illustrated in Fig. 1. The photodetection probability at a time is uniquely determined by the density operator of the photon field at the same time. This is the measurement action. However, whether or not a photoelectric conversion actually occurs is essentially uncertain due to quantum-statistical nature of light. On the other hand, an actual readout — no count or one-count — produces the density operator of the photon field at an infinitesimally later time via nonunitary state reduction. This is the measurement back action. The photon field thus develops in two different ways depending on the real-time measurement result, namely no count or one count. The crucial observation here is that the time-developed new density operator at an infinitesimally later time determines the photocount probability density at that time, but that, whether or not photoelectric conversion actually occurs is again uncertain due to the quantum-statistical nature of light. The conventional picture imposes quantum-mechanical state reduction only at the end of the measurement period, while continuous measurement imposes state reduction throughout the measurement period.

This paper is organized as follows. Section II gives a microscopic foundation of continuous measurement of photon number by extending conventional quantum theory of measurement that employs an operation-valued measure. Section III discusses effects of information readout or discarding on state reduction. Up to this point we will discuss continuous state reduction of a single quantum system. Section IV describes continuous state reduction of two quantum-mechanically correlated photon fields by continuous photodetection of one of the fields. The effects of measurement back action and quantum correlation on the state reduction of the other field will be high lighted. Section V presents a new scheme for quantum nondemolition measurement of photon number. Section VI presents novel phenomena that are unique to continuous measurement. First, we will report that a highly squeezed state oscillates in time between super- and sub-Poissonian photon statistics due to the back action of the photon number measurement. Second, we will propose a new scheme for generating a superposed state of two macroscopically distinguishable quantum states, namely, Schrödinger's cat state. Finally, Section VII summarizes the main results of this paper.

2 Microscopic foundation of continuous measurement of photon number

Suppose that the photodetector is coupled to the optical field in a closed cavity as shown in Fig. 2.[2] The output of the detector is a time sequence of photoelectric current pulses, each of which represents the detection of a photon. Such a photodetection process can be modeled as follows. Suppose that two-level atoms in the ground state are injected into the cavity one by one. Each atom interacts with the cavity field during the same time interval via the electric-dipole interaction. This interaction causes a unitary evolution of the atom-field system. After passing out of the cavity, the level of each atom is measured. An atom in the upper level corresponds to the one-count process, and the atom in the ground state corresponds to the no-count process. For each readout, $\rho_a^{(read)}$, the state of the system after the measurement of the apparatus is given by

$$\hat{\rho}_s(\Delta t) = \frac{T_{r_a}[\hat{\rho}_{s-a}(\Delta t)\hat{\rho}_a^{(read)}]}{T_{r_{s-a}}[\hat{\rho}_{s-a}(\Delta t)\hat{\rho}_a^{(read)}]}, \tag{1}$$

where subscripts a and s stand for atom and system, respectively. For such infinitesimal process, the state change of the system can be symbolically denoted as

$$\hat{\rho}_s(0) \rightarrow \hat{\rho}_s(\Delta t; X) = M[\hat{U}\hat{\rho}_s(0) \otimes \hat{\rho}_a(0)\hat{U}^\dagger], \tag{2}$$

where X denotes the readout. The time evolution of the system density operator during a finite time $t = N\Delta t$ is obtained by making N successive operation of this process.

$$\hat{\rho}_s(N\Delta t; X_1, X_2, \cdots, X_N) = M_N[\cdots[\hat{U}M_1\hat{\rho}_s(0) \otimes \hat{\rho}_a(0)\hat{U}^\dagger] \otimes \hat{\rho}_a(0)\hat{U}^\dagger] \otimes \cdots], \tag{3}$$

where M_n denotes operation corresponding to the readout X_n. The electric-dipole interaction can be described by the Jaynes-Cumming interaction Hamiltonian:

$$\hat{H}_{int} = \hbar g(\hat{a}\hat{\sigma}^\dagger + \hat{a}^\dagger\hat{\sigma}), \tag{4}$$

where $\hat{\sigma}$ is the level-lowering operator for the two-level atom. For each time duration, the initial state is prepared as

$$\hat{\rho}(t_0) = \hat{\rho}_f(t_0) \otimes |g><g|. \tag{5}$$

The evolution of the density operator for the total system can be calculated by a perturbation theory. For the one-count process, $\hat{\rho}_a^{(read)} = |e>_{aa}<e|$. Therefore we find that for one-count

process the density operator of the photon field changes as follows.

$$J\rho(t) = \lambda a\rho(t)a^{\dagger}. \tag{6}$$

For the no-count process, $\hat{\rho}_a^{(read)} = |g>_{aa}< g|$. Therefore the density operator of the photon field changes as follows.

$$S_{\tau}\rho(t) = e^{(i\omega+\frac{\lambda}{2})a^{\dagger}a\tau}\rho(t)e^{(i\omega-\frac{\lambda}{2})a^{\dagger}a\tau}. \tag{7}$$

These results coincide with those adovocated by Srinivas and Davies.[3] They postulated these relations as axioms from which they developed their theory of continuous photodetection. Here we have shown that their postulates are justified microscopically using a physical model of a photodetector.

3 Effects of information readout/discarding on state reduction

A. State evolution in referring measurement process

We refer to the process in which one photon is detected as a *one-count process.* As shown in the last section, the one-count process is described by a superoperator J as

$$J\rho(t) = \lambda a\rho(t)a^{\dagger}. \tag{8}$$

The operator J expresses the action of annihilating one photon from the photon field. The density operator of the post-measurement state is related to that of the pre-measurement state by

$$\rho(t^{\dagger}) = \frac{J\rho(t)}{\mathrm{Tr}[J\rho(t)]} = \frac{a\rho(t)a^{\dagger}}{< n(t) >}, \tag{9}$$

where t^{+} denotes a time infinetesimally later than t. Then the average photon number immediately after the one-count process defined by $< n(t^{\dagger}) >\equiv \mathrm{Tr}[\rho(t^{\dagger})a^{\dagger}a]$ is given by

$$< n(t^{+}) >=< n(t) > -1 + \frac{< [\Delta n(t)]^2 >}{< n(t) >}. \tag{10}$$

This result shows that the difference between the average photon numbers before and after the one-count process is not exactly equal to one, but it has an additional term depending on the photon number variance of the pre-measurement state.[4] This term is nothing but the Fano factor.

Let us examine some typical examples. The number state has no photon number fluctuations. Therefore the average photon number decreases exactly by one in the one-count process. On the other hand, the Poissonian state such as a coherent state has a finite photon number variance which is equal to the average photon number. Therefore the average photon number does not change in spite of the fact that one photon was annihilated from the photon field. Presumably, the most unexpected result is for the chaotic state. In this case, the average photon number is doubled upon a single photon being detected. But why?

The apparent paradoxes are resolved if we take into account the effects of continuous measurement and its back action on the photon field. For simplicity, let us assume that the initial state is a mixture of the vacuum state and a number state:

$$\rho(0) = \frac{1}{2}(|0><0| + |100><100|). \tag{11}$$

Then the initial average photon number is 50. When one photon is detected for this state, the possibility that the initial state was the vacuum state suddenly vanishes. Thus we can conclude that the initial state was $|100><100|$. Since one photon has been detected just now, we can say that the present state is $|99><99|$ with the average photon number increased by 49. It is almost doubled.

The thermal state has a large probability of being in the vacuum state. However, once one photon is detected, the probability of the vacuum state suddenly vanishes. Therefore when we renomalize the density operator, this vanishing probability is redistributed over the other states, causing an increase in the average photon number.

Next, let us examine the no-count process. We refer to the process in which no photons are detected as a no-count process. As shown in the Sec. II, the no-count process is described by a superoperator S_τ as

$$S_\tau\rho(t) = e^{(i\omega+\frac{\lambda}{2})a^\dagger a\tau}\rho(t)e^{(i\omega-\frac{\lambda}{2})a^\dagger a\tau}. \tag{12}$$

The density operator of the post-measurement state is expressed in terms of that of the pre-measurement state by

$$\rho(t+\tau) = \frac{S_\tau\rho(t)}{\mathrm{Tr}[S_\tau\rho(t)]} = \frac{e^{-(i\omega+\frac{\lambda}{2})a^\dagger a\tau}\rho(t)e^{(i\omega-\frac{\lambda}{2})a^\dagger a\tau}}{\mathrm{Tr}[\rho(t)e^{-\lambda a^\dagger a\tau}]}. \tag{13}$$

Since the operator S_τ includes only the number operator, it does neither create nor annihilate photons in the field. Nevertheless the statistical properties of the observed photon field change in time in a nonunitary way due to the field -detector coupling represented by λ. After a simple algebraic manipulation, we can obtain the following result:

$$\frac{d}{dt} < n(t) >= -\lambda < [\Delta n(t)]^2 > . \tag{14}$$

Thus we find that the average photon number decreases in time at a rate proportional to the photon number variance. Therefore, it does not change for number state, but decays for all other states, although no photons are detected actually. But why?

To be in accordance with the fact that we have not detected any photons for a long time, we must modify our knowledge of the initial photon statistics, so that the probability of the vacuum state is increased and probabilities of the other number states are decreased. Such a modification results in a decrease in the average photon number, even though no photon has actually been extraced from the photon field. Generalizing the above discussion, it is easy to see that the density operator is modified every moment according to the results of continuous measurements.

Figure 3 illustrates the time evolution of the average photon number for initially (a) a number state, (b) a sub-Poissonian squeezed state, and (c) a thermal state. The times τ_1, τ_2, $\cdots$ indicate the times when photons were detected. The dashed curves correspond to the initially coherent state. When the initial state is a number state, the average photon number does not change under the no-count process, and decreases by 1 for every one-count process. When the initial state is a sub-Poissonian squeezed state, the average photon number decreases more slowly than that of the coherent state under the no-count process, and it decreases less than 1 for the one-count process. When the initial state is a thermal state, the average photon number decreases fastest under the no-count process, but it *increases* for the one-count process in contrast to the other states.

B. State evolution in non-referring measurement process

Thus far we have examined nonunitary time evolution of the photon field in the referring measurement process. That is, we read out all available information concerning no count or one count, and use the readout information to renormalize the photon density operator. On the other hand, it is completely at our choice wherther we use the obtained information or discard it. What happens to the photon field if we discard all information about photocounts except for knowledge that the detector is active ? We refer to such a process as the *non-referring* *measurement process* (NMP).[1]

Let us introduce a superoperator T_τ such that it describes the time development of the density operator in the NMP. Since we do not refer to the result of measurement, T_τ must be a statistical summation of the one-count and no-count processes.

$$T_{dt}\rho(t) = J\rho(t)dt + S_{dt}\rho(t). \tag{15}$$

This equation leads to a differetial equation for the density operator of the photon field in the NMP:

$$\frac{d}{dt}\rho(t) = \lambda a\rho(t)a^\dagger - [i\omega + \frac{\lambda}{2}]a^\dagger a\rho(t) + [i\omega - \frac{\lambda}{2}]\rho(t)a^\dagger a. \tag{16}$$

Note that this differetial equation is identical to the stochastic master equation in the Markoffian process. This should be so because in the NMP the past events do not affect the future. This operator differential equation can be integrated to give[1,4]

$$\rho(t+\tau) = \sum_{k=0}^{\infty} \frac{(1-e^{\lambda\tau})^k}{k!} \exp[-(i\omega + \frac{\lambda}{2})a^\dagger a\tau]a^k\rho(t)(a^\dagger)^k \exp[(i\omega - \frac{\lambda}{2})a^\dagger a\tau]. \tag{17}$$

From this equation we can calculate the time evolution of photon statistics in the NMP. For example, the average photon number decreases exponentially in time. This is because we discard all readout information so that the photodetector comes to play the simple role of a linear absorber. The time development of the Fano factor shows that the original photon statistics, represented by $F(t)$, lose their feature as time proceeds. No matter what the initial statistics are, they àpproach the Poissonian.

To elucidate the meaning of the NMP, let us consider the initially number state. The density operator after the NMP of duration τ is given by

$$\rho(\tau) = \sum_{m=0}^{n_0} \binom{n_0}{m}(1-e^{-\lambda\tau})^m(e^{-\lambda\tau})^{n_0-m}|n_0 - m >< n_0 - m|. \tag{18}$$

This can be understood as follows: our knowledge that the detector is active leads to a conclusion that some of the initial n_0 photons can be detected by a photodetector with probability $p = 1 - e^{-\lambda\tau}$. The coefficient in the summand of the above equation gives the probability of m out of n_0 photons being detected with probability p. However, since we do not know the number of photons that are actually detected, the density operator after this measurement process falls into a mixture of all possible numbers. Consequently, the Fano factor increases toward unity as time progresses. In contrast, for the RMP, we do know the number of detected photons. Therefore the post-measurement density operator does not fall into a mixture but remains a pure state.

Figure 4 shows time development of the Fano factor in the non-referring and referring processes. In the non-referring process all states except for the coherent state collapses into a mixture. On the other hand, in the referring measurement process, all initially pure states remain pure states, although their statistics, in general, change in time.

C. Conservation of energy in continuous measurement

We have shown that the average photon number *increases* in the one-count process for a super-Poissonian state, while it *decreases* in the no-count process for all states except number state. We identify the underlying physics of this effect as the vanishing or enhancing probability of the vacuum state and the associated renormalization of the density operator. A natural question yet arises: How is the energy conserved in the continuous measurement process ? Now we will discuss this problem.

Suppose that the measurement process began at $t = 0$ and m photons have been detected by the time $t = T$. Then the density operator immediately after $t = T$ is given by[1]

$$\rho_m^{QPN}(T) = \frac{\exp[-(i\omega + \frac{\lambda}{2})a^\dagger aT]a^m \rho(0)(a^\dagger)^m \exp[(i\omega - \frac{\lambda}{2})a^\dagger aT]}{\mathrm{Tr}[\tau(0)(a^\dagger)^m \exp(-\lambda a^\dagger aT)a^m]}, \tag{19}$$

and hence the average photon number of the remaining field is calculated as[4]

$$< n(T) >_m = \mathrm{Tr}[\rho_m^{QPN}(T)a^\dagger a] = \exp(-\lambda T)\frac{\mathrm{Tr}[\rho(0)(a^\dagger)^{m+1}\exp(-\lambda a^\dagger aT)a^{m+1}}{\mathrm{Tr}[\rho(0)(a^\dagger)^m \exp(-\lambda a^\dagger aT)a^m}, \tag{20}$$

where the subscript m in $< n(T) >_m$ indicates the number of detected photons up to time T. It is clear that the sum, $m+ < n(T) >_m$, is not, in general, equal to the average photon number of the initial state $n_0 \equiv \mathrm{Tr}[\rho(0)a^\dagger a]$:

$$m+ < n(T) >_m \neq n_0. \tag{21}$$

This is because m is a result of a single measurement. Since the conservation law in quantum mechanics holds true in the ensemble-average context, the conserved quantity is the ensemble average of the sum with respect to m. It can be shown that the ensemble averaged quantity certainly equals n_0:[4]

$$\sum_{m=0}^{\infty} P(m;0,T)[m+ < n(T) >_m] = 0. \tag{22}$$

4 Continuous state reduction of correlated photon fields in photodetection process

Up to this point, we have discussed continuous state reduction of a single quantum system. Now, we will describe continuous state reduction of two quantum-mechanically correlated photon fields.[5]

Suppose that two photon fields, which we will from now on refer to as signal and idler fields, interact with each other (see Fig. 5). This interaction is assumed to establish a quantum correlation between these two fields. Then we start a photon counting experiment for the idler field. The idler photons are destructively measured by a photodetector one by one. Therefore, the idler field reduces towards the vacuum state. The nonunitary state evolution of the idler field is exactly the same as what we have examined thus far. The main concern here is how the signal state reduction is caused by the back action of the idler measurement through the establised quantum correlation.

Since the one-count process annihilates one photon in the photon field, the state change in the one-count process can be obtained by operating the annihilation operator from left and the creation operator from right. The difference between the average photon numbers of the idler field before and after the one-count process is similar to what we have discussed for the case of a single quantum system: The difference does not exactly equal one but it has an additional term called the Fano factor. On the other hand, time development of the signal photon number depends on the quantum correlation between the signal and idler photon numbers of the pre-measurement state (see Fig. 6 A).

Also, in the no-count process, the average photon number of the idler field decays at a rate proportional to the photon number variance of its own. In contrast, the average photon number of the signal field decreases in time at a rate proportional to the photon number covariance between the two fields (see Fig. 6 B). Thus we find that the idler photon statistics evolves according only to the measurement back action, but that the signal photon statistics is changed only through quantum correlation.

It is well known that the signal and idler fields generated in the process of parametric down conversion have a perfect photon number correlation if the input signal and idler fields are in the vacuum state. This is the Manley-Rowe relation. A question arises whether or not this perfect quantum correlation is deteriorated by the destructive photocounting measurement. A detailed calculation gives the density operator of the total system after m idler photons were detected during time T (see Fig. 6 C). From this result we find that the average photon numbers in the signal and idler fields differ exactly by the number of detected photons. Furthermore, the photon number noises are still correlated perfectly. To put it differently, the Manley-Rowe relation is preserved in the *destructive* photodetection process if we retain information about the number of detected photons.

Figure 7 illustrates time development of average photon numbers of the signal and idler fields in the referring measurement process. One-count processes are assumed to occur at times τ_1, τ_2, etc. We can see that the idler photon statistics are reducing towards the vacuum state. In contrast, the signal photon statistics are continuously reducing towards the number state.

Figure 8 illustrates time developments of average photon numbers of the signal and idler fields in the non-referring measurement process, namely, the time developments when we discard readout information. The dashed curves show the corresponding ones in the referring measurent process. We can see that in this case the photodetector plays a simple role of a linear absorber for the idler fields, while it does not affect the signal field at all because in this case quantum correlation can play no role.

Figure 9 shows time developments of the Fano factor in the referring and non-referring processes. The idler field reduces towards the vacuum state for both processes, although the way state approaches the vacuum state are different. In contrast, the signal field reduces towards the number state in the referring measurement process because of the Manley-Rowe relation, while it remains unchanged in the non-referring measurement process.

5 Quantum nondemolition photon counting

Now we propose a new scheme for quantum nondemolition (QND) measurement of photon number. The difference between the QND photon counting and the usual photon counting lies in the fact that the usual photon-counting process ends when all photons in the cavity are counted and the state finally reduces to the vacuum state. On the other hand, the QND photon counting never ends because photons are not absorbed owing to the nondemolition nature of the process, and the state finally reduces not to the vacuum state but to a number state. We want to see how an arbitrary initial state continuously reduces towards a number state. We also want to know what state the same initial state reduces to if we discard the readout information.

The model Hamiltonian is written as

$$\hat{H}_{int} = \hbar g \hat{a}^{\dagger}\hat{a}(\hat{\sigma} + \hat{\sigma}^{\dagger}), \tag{23}$$

where $\hat{\sigma}$ is a transition operator for a bi-stable quantum device. For example, a molecule having right helicity and left helicity can be such a device. The molecule changes its state from left to right or vice versa by absorbing and successively emitting a photon. For simplicity, we will call such a device as an "atom".

It is clear that with this Hamiltonian the photon-number measurement meets the following requirements for QND measurement.

$$1.\ [\hat{A}_s(0), \hat{A}_s(t)] = 0, \tag{24}$$

$$2.\ [\hat{H}_{int}, \hat{A}_s] = 0, \tag{25}$$

$$3.\ [\hat{H}_{int}, \hat{A}_p] \neq 0, \quad \text{and} \tag{26}$$

$$4.\ \hat{H}_{int} \text{ should be a function of } \hat{A}_s. \tag{27}$$

We assume that the bi-stable atoms are initially prepared in a determined state, and that they are injected into the cavity field one by one. Each atom passes through the cavity within the same time duration and interacts with the field via the interation Hamiltonian $H_{\rm int}$. After passing through the field, the state of each atom is measured. The atom in the original state corresponds to the no-count process, and the atom in the other state corresponds to the one-count process.

Let us first examine what state the initial state reduces to if we do not refer to the result of measurement. In this case it can be shown that the density operator obeys the differential equation.

$$\frac{d}{dt}\hat{\rho} = -\frac{\lambda}{2}\left[\hat{\rho}_f \hat{n}^2 + \hat{n}^2 \hat{\rho}_f\right], \tag{28}$$

where $\lambda \equiv g^2 \Delta t$. This equation can be immediately solved to give the following solution.

$$\hat{\rho}_{mn}(t) = e^{-i\omega(m-n)} \exp\left[-\frac{\lambda}{2}(m-n)^2 t\right]. \tag{29}$$

It is easy to see that after a long time diagonal matrix elements remain their initial value, whereas off-diagonal elements vanish. Therefore we find that the density operator of the photon field is being continuously diagonalized if we discard the readout information.

Figure 10 shows the continuous diagonalization of an initially coherent state in the non-referrring measurement process.

Figure 11 shows time developments of the Fano factor for various initial quantum states when we renormalize the density operator in *real time* according to the readout information. We can see that all states continuously reduce to the number state, although the way each state reduces depends strongly on the initial photon statistics.

6 Novel phenomena unique to continuous measurement

This section presents two novel phenomena that are unique to continuous measurement.

6.1 Measurement-induced oscillations of a highly squeezed state between super- and sub-Poissonian photon statistics

First, we report that a highly squeezed state oscillates in time between super- and sub-Poissonian photon statistics due to the back action of the photon number measurement.[6] A squeezed state is generated from a coherent state by a two-photon process. Mathematically this can be performed by first displacing the vacuum state by α, and then operating on it a squeezing operator $S(r)$,

$$\begin{aligned} |\alpha, r> &\equiv \exp\left\{\frac{r}{2}\left[a^2-(a^\dagger)^2\right]\right\}|\alpha> \\ &= \frac{\exp\left[-\frac{|\alpha|^2}{2}+\frac{\alpha^2}{2}\tanh r\right]}{\sqrt{\cosh r}}\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}\left(\frac{\tanh r}{2}\right)^{n/2}\mathrm{H}_n\left(\frac{\alpha}{\sqrt{\sinh 2r}}\right)|n>, \end{aligned} \tag{30}$$

where r is a squeezing parameter and $\mathrm{H}_n(z)$ is the nth Hermite polynomial defined as

$$\mathrm{H}_n(z)=\sum_{m=0}^{[n/2]}\frac{(-1)^m n!}{m!(n-2m)!}(2z)^{n-2m}. \tag{31}$$

A highly squeezed state is characterized with a large squeezing parameter r. This state, in general, exhibit a super-Poissonian character with a large average photon number. The density operator of the squeezed state becomes

$$\begin{aligned} \rho(0) &= \frac{\exp\left[\alpha^2(\tanh r-1)\right]}{\sqrt{\cosh r}} \\ &\times \sum_{n,m=0}^{\infty}\frac{1}{\sqrt{m!n!}}\left(\frac{\tanh r}{2}\right)^{(m+n)/2}\mathrm{H}_m\left(\frac{\alpha}{\sqrt{\sinh 2r}}\right)\mathrm{H}_n\left(\frac{\alpha}{\sqrt{\sinh 2r}}\right)|m><n|, \end{aligned} \tag{32}$$

where we assume that α is real because the phase of the coherent state does not affect the following discussion. Time development of an initially squeezed state can be calculated as follows.[4]

$$\rho_m^{\mathrm{QPF}}(\tau) = \frac{1}{N}\sum_{n,k=0}^{\infty}\frac{1}{\sqrt{k!n!}}\left(\frac{\tanh r}{2}\right)^{(k+n)/2}\exp(-i\Omega k\tau + i\Omega^* n\tau) \times \mathrm{H}_{k+m}\left(\frac{\alpha}{\sqrt{\sinh 2r}}\right)\mathrm{H}_{k+n}\left(\frac{\alpha}{\sqrt{\sinh 2r}}\right)|k><n|, \tag{33}$$

where

$$N \equiv \frac{d^m}{dz^m}\left[\exp\left(\frac{2z}{1+z}\frac{\alpha^2}{\sinh 2r}\right)(1-z^2)^{-1/2}\right]_{z=e^{-\lambda\tau}\tanh r}, \tag{34}$$

and $\Omega = \omega - i\lambda/2$. Here m denotes the number of detected photons until time t.

Figure 12 shows the time development of an initially highly squeezed state. It is noted that the Fano factor oscillates in time between super- and sub-Poissonian regimes. This presents a sharp contrast to the case for an initially sub-Poissonian squeezed state, as shown in Fig. 4. In this case, the Fano factor monotonically increases towards unity. A monotonic increase or decrease of the Fano factor is usually expected in any destructive measurement like a photon-counting experiment. If so, why such novel oscillations occur for a highly squeezed state ?

This paradox can be resolved if we recall that we are considering a referring measurement process in which the density operator is renormalized in *real time* according to the readout information. The no-count process continues up to time τ_1. Such a no-count process allows of two interpretations. One is that the pre-measurement state is the vacuum state. The other is that the pre-measurement state is antibunched. Since we certainly know that the pre-measurement state has a large average photon number, we can remove the first possibility. It can be shown that if a state exhibits antibunching for some time, the state also shows a sub-Poissonian character in the same time domain.[7] Therefore, the Fano factor moves towards the sub-Poissonian regime. However, once one photon is detected before long, then this information enhances the possibilities of number states with large eigenvalues. For quadrature-amplitude squeezed states as we are considering, this leads to the conclusion that the initial state exhibits a super-Poissonian character. Therefore, the Fano-factor suddenly jumps above unity.

6.2 Generation of Shrödinger's cat-like state by continuous photodetection

The underlying physics of these novel Fano-factor oscillations can be seen from a different point of view. First of all, we note the following fact: the time-developed density operator satisfies the idempotency condition.

$$\left[\hat{\rho}^{QPF}(t)\right]^2 = \hat{\rho}^{QPF}(t) \tag{35}$$

That is, an initially squeezed state evolves nonunitarily in the referring measurement process, but it remains a pure state, even though photon counting is a second-kind measurement.

Figure 13 shows the time development of the density operator of an initially squeezed state in the quasi-probability density representation. We can see that the quasi-probability distribution bifurcates almost completely when two photons are detected. Since we have shown that this state is a pure state, we can conclude that this state is a superposed state of two macroscopically distinguishable state, namely, Schrödinger's cat state.[8]

7 Conclusion

We have developed general formulas for continuous photodetection processes. These formulas describe nonunitary time evolution of the photon field under continuous measurement of photon number. We have shown the way the state reduces towards the vacuum state depends strongly on the initial photon statistics. In particular, we have found that the average photon number after the one-count process increases when the premeasurement state is super-Poissonian. We identify the physical origins of this effect as the vanishing probability of the vacuum state and the associated renormalization of the density operator in the one-count process. We have introduced the non-referring measurement process and by comparing it with the referring measurement process we have discussed the effect of discarding observable information of the system on its state reduction. We have derived general formulas which show that the initially pure state, in general, collapses into a mixed state.

In the quantum theory of measurement process, the nonúnitary process due to the measurement is usually considered to be an instantaneous process. The evolution of the state is obtained using the operation-valued measure. The present analysis enables us to trace the time evolution of the photon field in real time as the state reduces towards the vacuum state. In this sense, the present analysis extends the conventional quantum theory of measurement. However, the present analysis is not a kind of measurement theory which "explains" why the wavefunction collapses; it is a continuation of infinitesimal nonunitary processes (one-count and no-count-processes) which are postulated.

References

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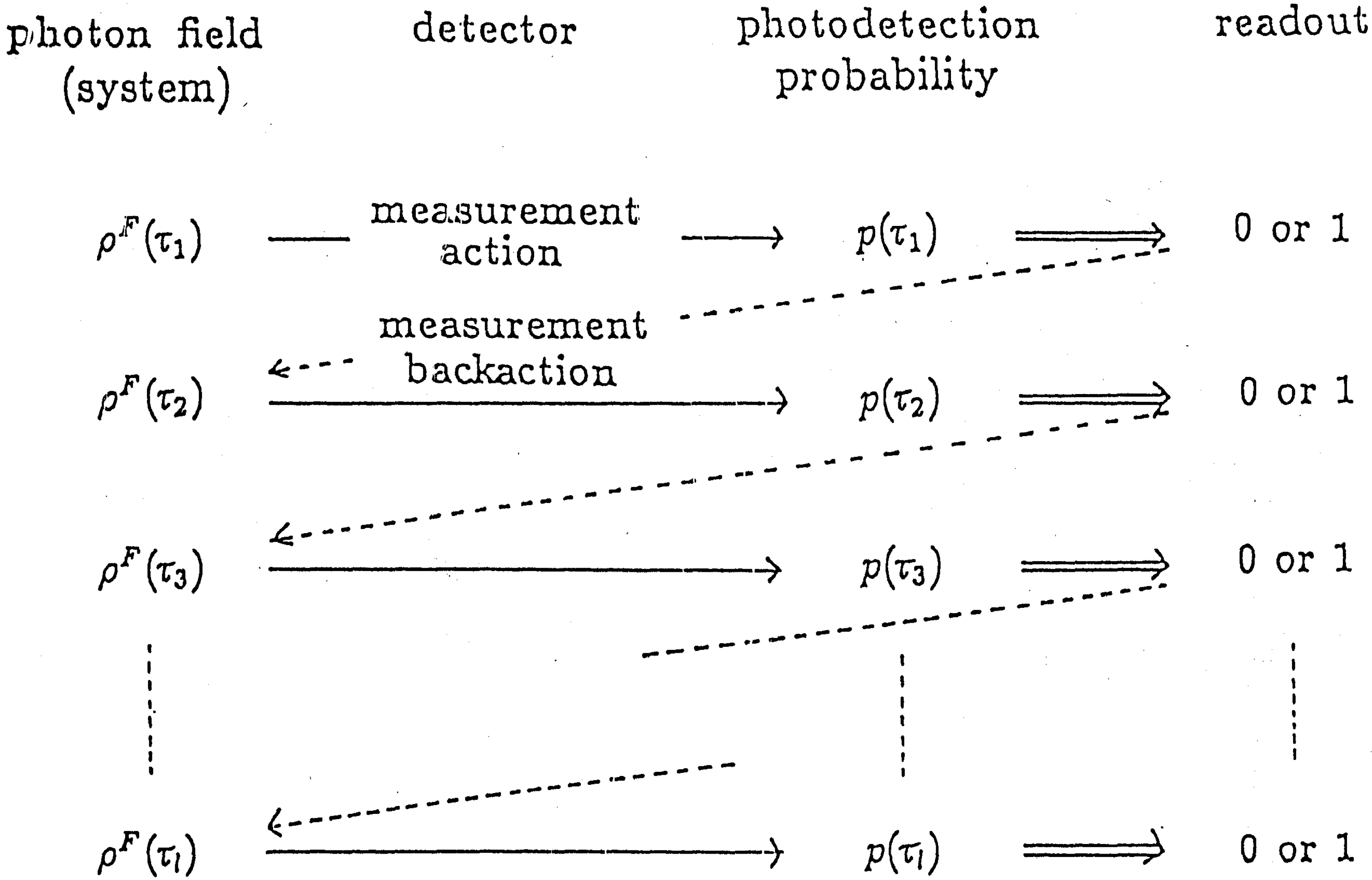

photon field
(system)
detector
photodetection
probability
readout
$\rho^F(\tau_1)$
measurement
action
$p(\tau_1)$
0 or 1
measurement
backaction
$\rho^F(\tau_2)$
$p(\tau_2)$
0 or 1
$\rho^F(\tau_3)$
$p(\tau_3)$
0 or 1
$\rho^F(\tau_l)$
$p(\tau_l)$
0 or 1

Fig.1

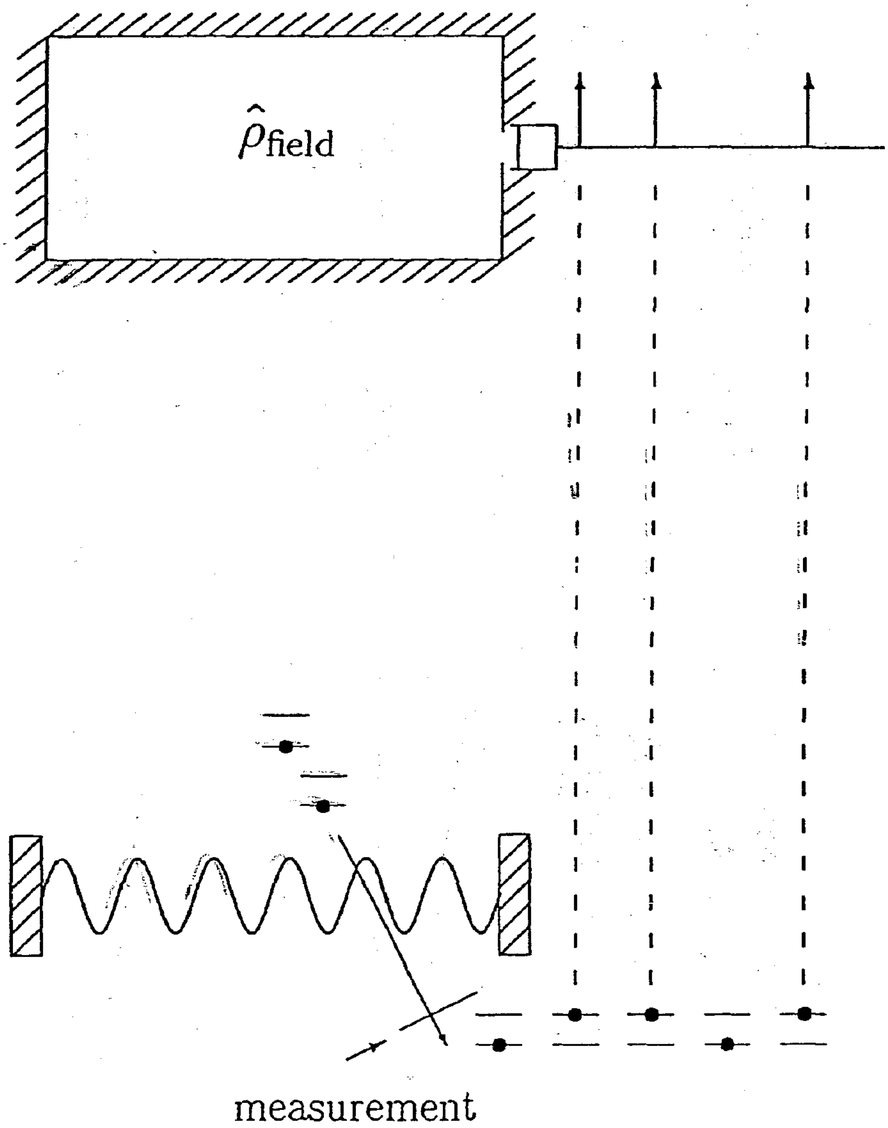


Fig.2

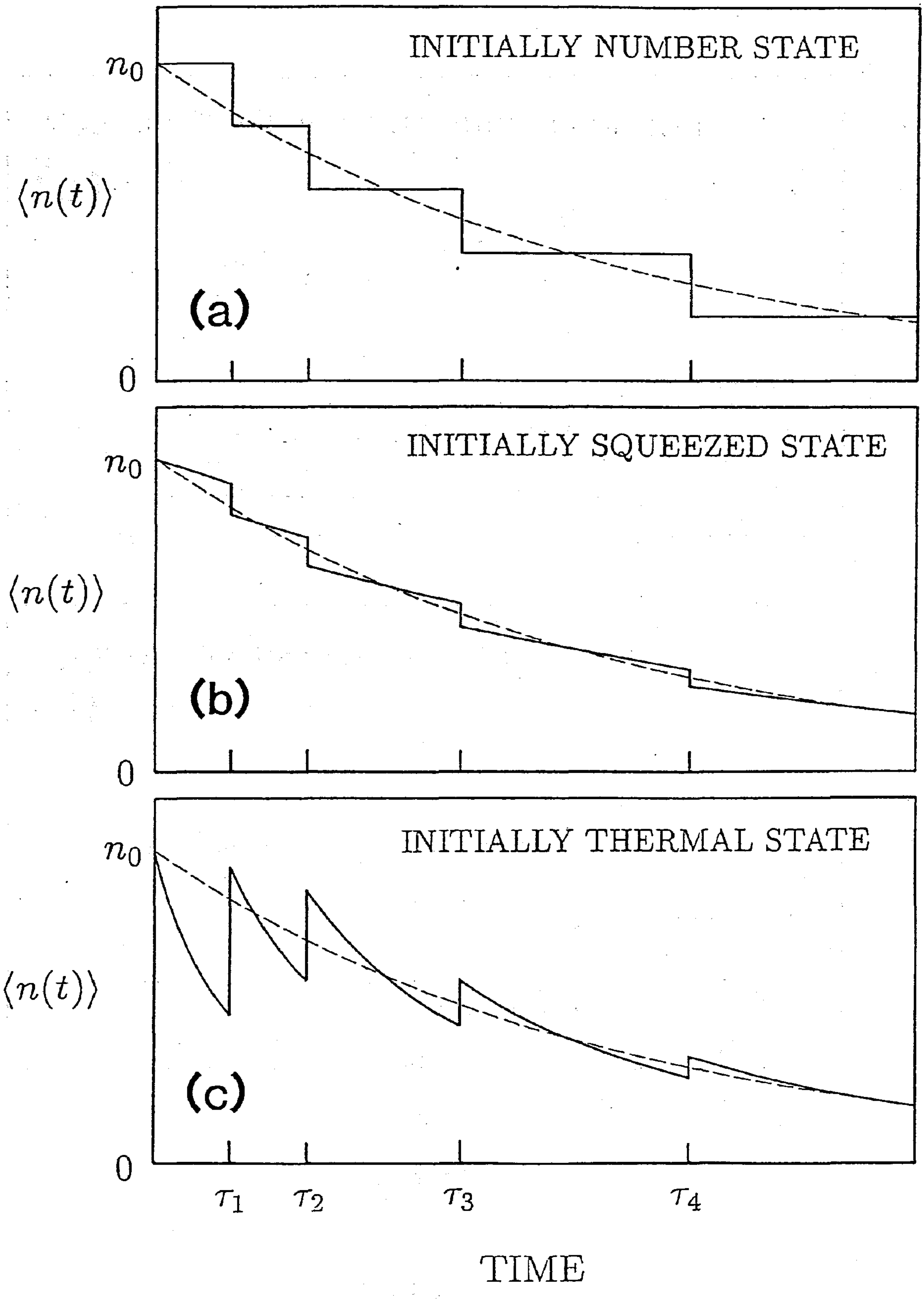
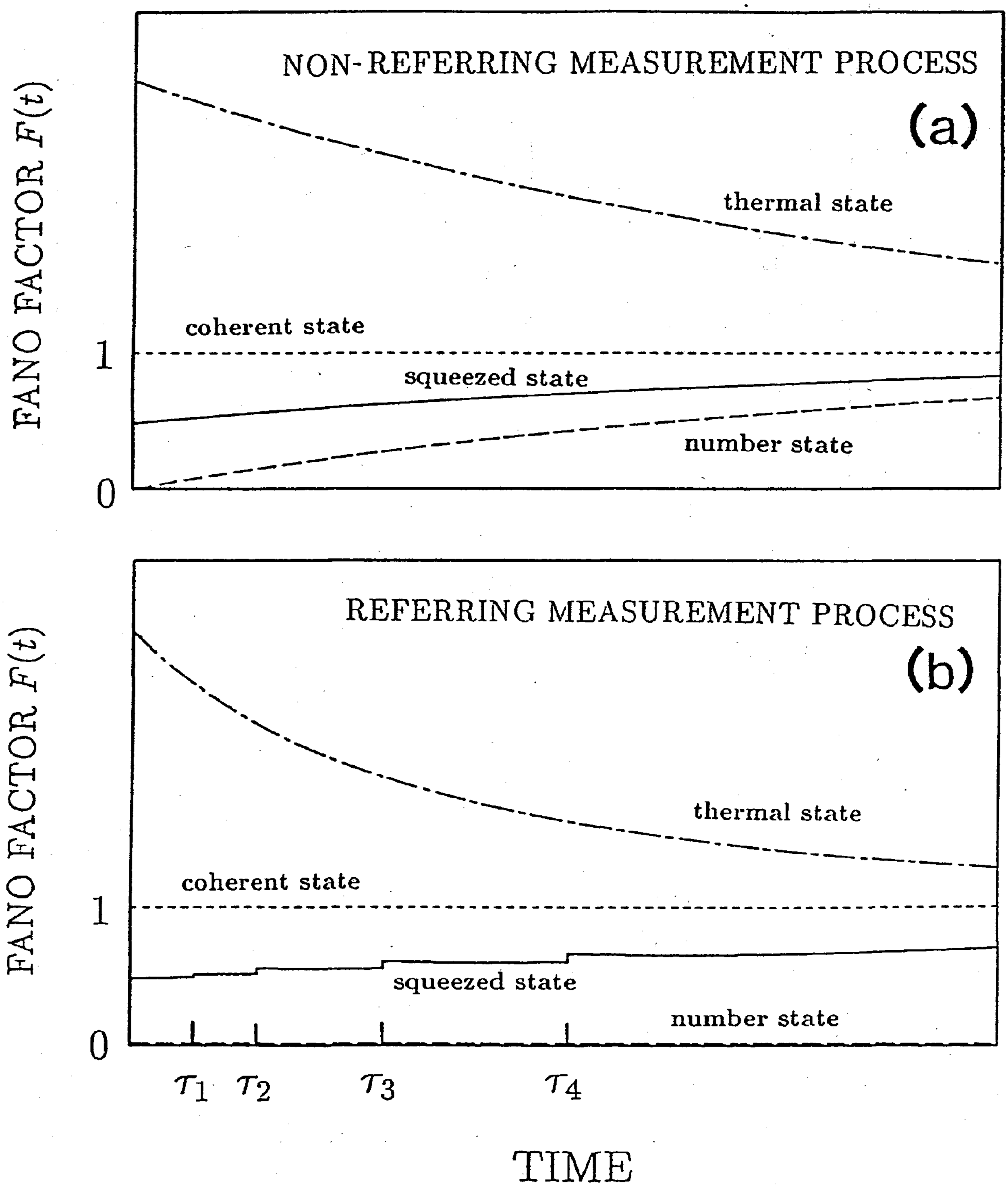

INITIALLY NUMBER STATE
n_0
$\langle n(t) \rangle$
(a)
0
INITIALLY SQUEEZED STATE
n_0
$\langle n(t) \rangle$
(b)
0
INITIALLY THERMAL STATE
n_0
$\langle n(t) \rangle$
(c)
0
τ_1
τ_2
τ_3
τ_4
TIME

Fig.3



NON-REFERRING MEASUREMENT PROCESS
(a)
FANO FACTOR $F(t)$
thermal state
coherent state
1
squeezed state
number state
0
REFERRING MEASUREMENT PROCESS
(b)
FANO FACTOR $F(t)$
thermal state
coherent state
1
squeezed state
number state
0
τ_1 τ_2 τ_3 τ_4
TIME

Fig.4

Continuous state reduciton of correlated photon fields

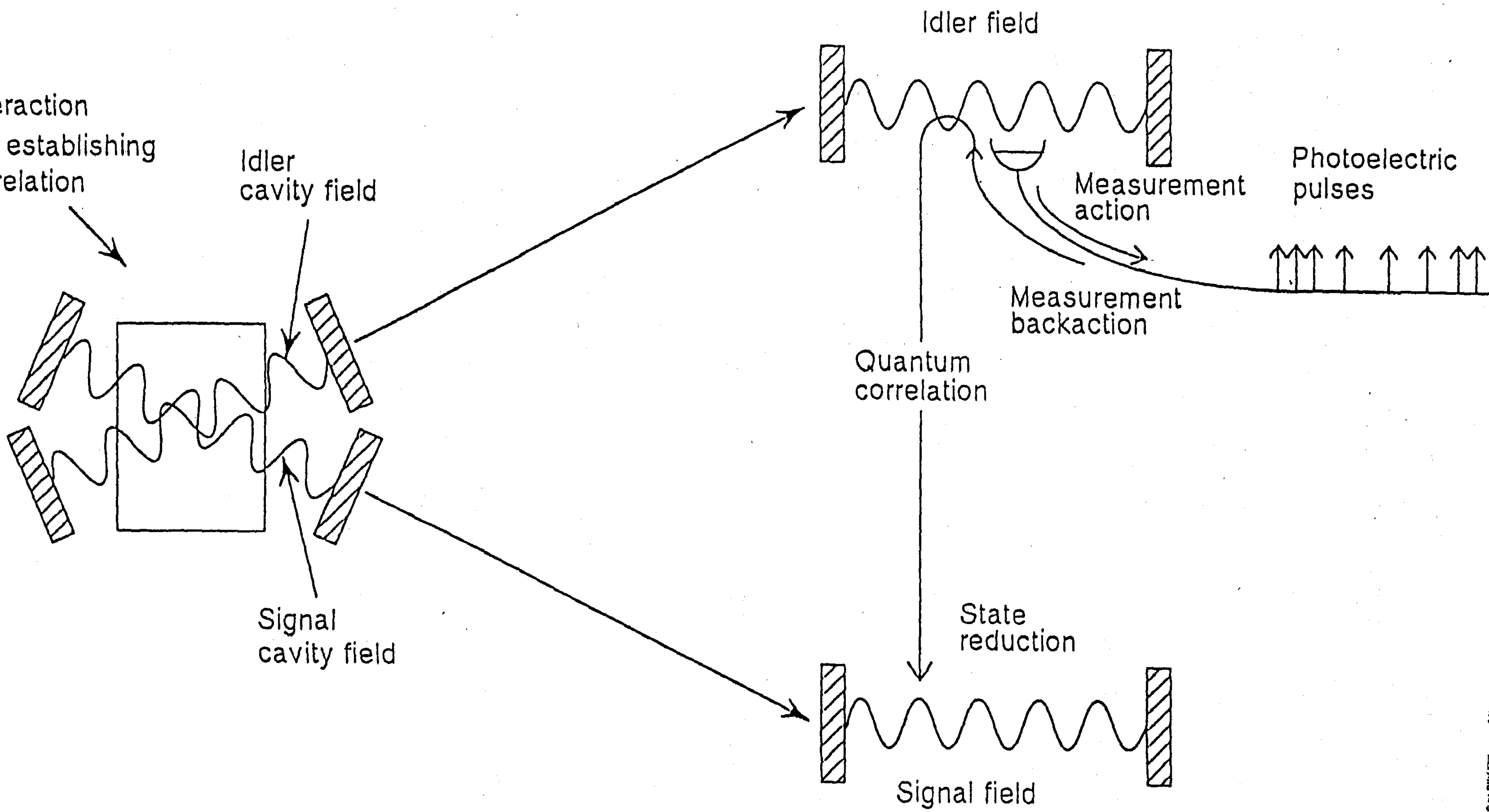


Fig.5

Continuous state reduction of correlated photon fields

A. One-count process

$$J^{(\mathrm{i})}\rho(t) \equiv \lambda a_{\mathrm{i}}\rho(t)a_{\mathrm{i}}^{\dagger} \qquad \rho(t^{+}) = \frac{J^{(\mathrm{i})}\rho(t)}{\mathrm{Tr}\left[J^{(\mathrm{i})}\rho(t)\right]} = \frac{a_{\mathrm{i}}\rho(t)a_{\mathrm{i}}^{\dagger}}{\langle n_{\mathrm{i}}(t)\rangle}$$

$$\langle n_{\mathrm{i}}(t^{+})\rangle = \langle n_{\mathrm{i}}(t)\rangle - 1 + \frac{\langle[\Delta n_{\mathrm{i}}(t)]^2\rangle}{\langle n_{\mathrm{i}}(t)\rangle} \qquad \langle n_{\mathrm{s}}(t^{+})\rangle = \frac{\langle n_{\mathrm{s}}(t)n_{\mathrm{i}}(t)\rangle}{\langle n_{\mathrm{i}}(t)\rangle}$$

B. No-count process

$$S_{\tau}^{(\mathrm{i})}\rho(t) = e^{Y\tau}\rho(t)e^{Y^{\dagger}\tau} \qquad \rho(t+\tau) = \frac{S_{\tau}^{(\mathrm{i})}\rho(t)}{\mathrm{Tr}\left[S_{\tau}^{(\mathrm{i})}\rho(t)\right]}$$

$$Y = -\left(i\omega_{\mathrm{i}} + \frac{\lambda}{2}\right)a_{\mathrm{i}}^{\dagger}a_{\mathrm{i}} - i\omega_{\mathrm{s}}a_{\mathrm{s}}^{\dagger}a_{\mathrm{s}}$$

$$\frac{d}{dt}\langle n_{\mathrm{i}}(t)\rangle = -\lambda\langle[\Delta n_{\mathrm{i}}(t)]^2\rangle \qquad \frac{d}{dt}\langle n_{\mathrm{s}}(t)\rangle = -\lambda\langle\Delta n_{\mathrm{i}}(t)\Delta n_{\mathrm{s}}(t)\rangle$$

C. Generalized Manley-Rowe relation

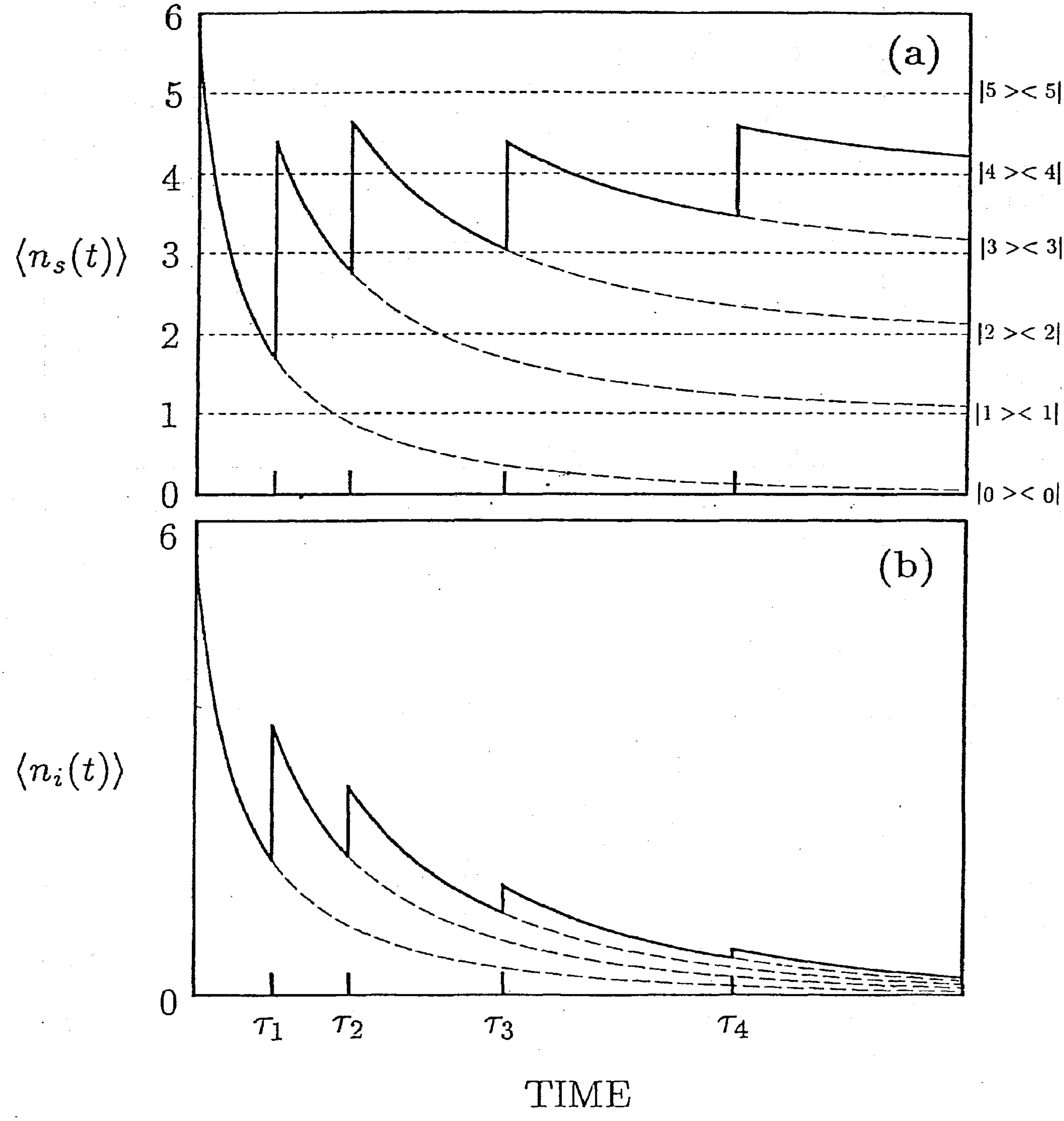
$$\left.\begin{aligned}&\langle n_{\mathrm{s}}(t_0)\rangle - \langle n_{\mathrm{i}}(t_0)\rangle = 0\\&\langle\{\Delta[n_{\mathrm{s}}(t_0) - n_{\mathrm{i}}(t_0)]\}^2\rangle = 0\end{aligned}\right\} \qquad \begin{aligned}&\text{Manley-Rowe relation}\\&n_{\mathrm{s}}(t_0) - n_{\mathrm{i}}(t_0) = n_{\mathrm{s}}(0) - n_{\mathrm{i}}(0)\end{aligned}$$

$$\rho_m^{\mathrm{QPF}}(T) = \left[1 - e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)\right]^{m+1}\sum_{k,l=0}^{\infty}\sqrt{\binom{k+m}{k}\binom{l+m}{l}}\,\alpha^k(\alpha^*)^l$$
$$\times\; e^{(\beta_s+\beta_i)(k-l)-\frac{\lambda}{2}(T-t_0)(k+l)}|k+m\rangle_{s\,s}\langle l+m|\otimes|k\rangle_{i\,i}\langle l|.$$

$$\left.\begin{aligned}&\langle n_{\mathrm{s}}(T)\rangle_m - \langle n_{\mathrm{i}}(T)\rangle_m = m\\&\langle\{\Delta[n_{\mathrm{s}}(T) - n_{\mathrm{i}}(T)]\}^2\rangle_m = 0\end{aligned}\right\} \qquad \begin{aligned}&\text{generalized Manley-Rowe relation}\\&n_{\mathrm{s}}(T) - n_{\mathrm{i}}(T) - m = n_{\mathrm{s}}(0) - n_{\mathrm{i}}(0)\end{aligned}$$

The Manley-Rowe relation is preserved in the *destructive* photodetection process if we retain information about the number of detected photons.

Fig.6



6
5
4
3
2
1
0
(a)
$\langle n_s(t) \rangle$
$|5><5|$
$|4><4|$
$|3><3|$
$|2><2|$
$|1><1|$
$|0><0|$
6
0
(b)
$\langle n_i(t) \rangle$
τ_1
τ_2
τ_3
τ_4
TIME

Fig.7

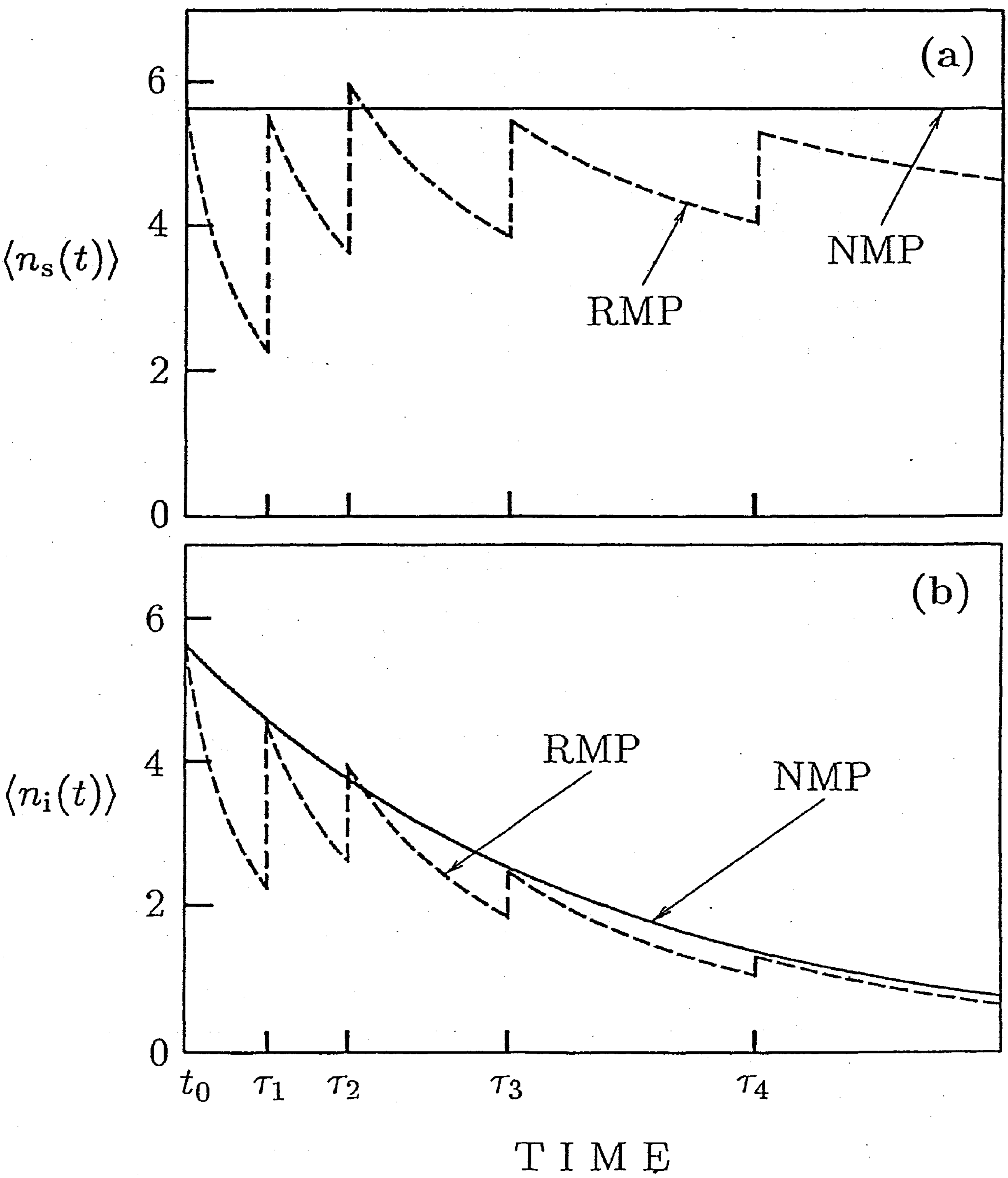

(a)
6
4
2
0
$\langle n_s(t) \rangle$
RMP
NMP
(b)
6
4
2
0
$\langle n_i(t) \rangle$
RMP
NMP
t_0
τ_1
τ_2
τ_3
τ_4
TIME

Fig.8

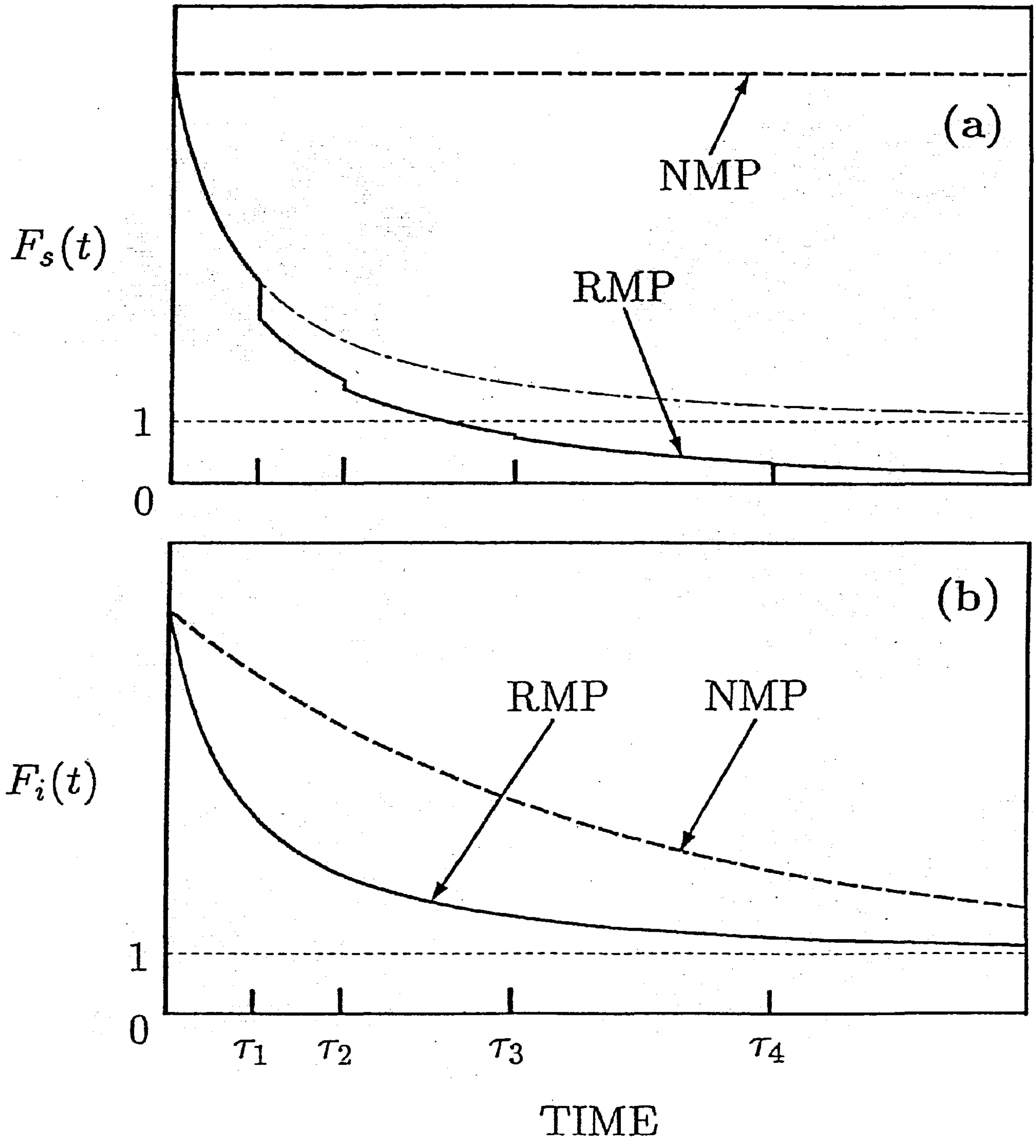


Fig.9

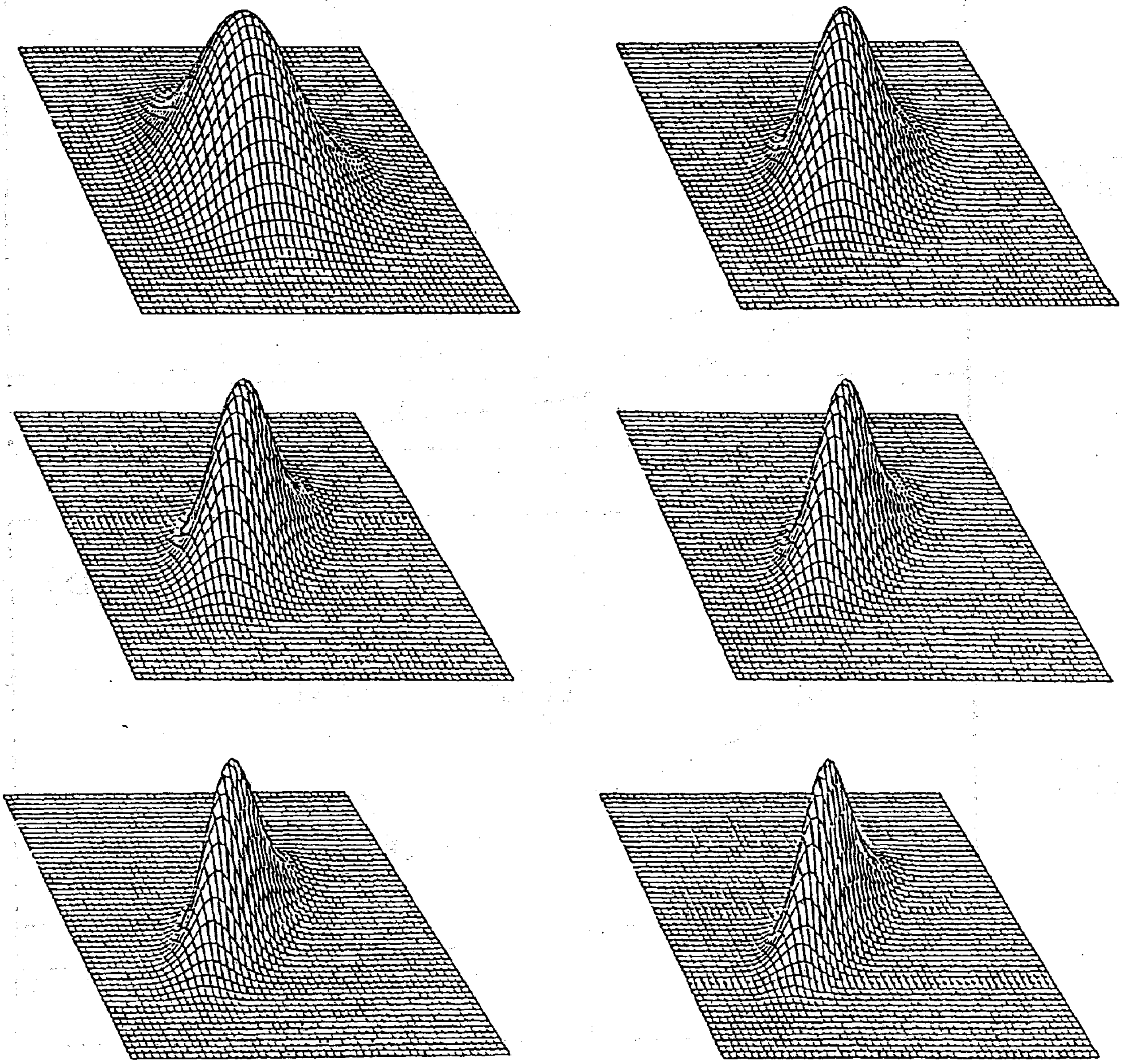


Fig.10

Continuous quantum nondemolition photon counting
Referring measurement process

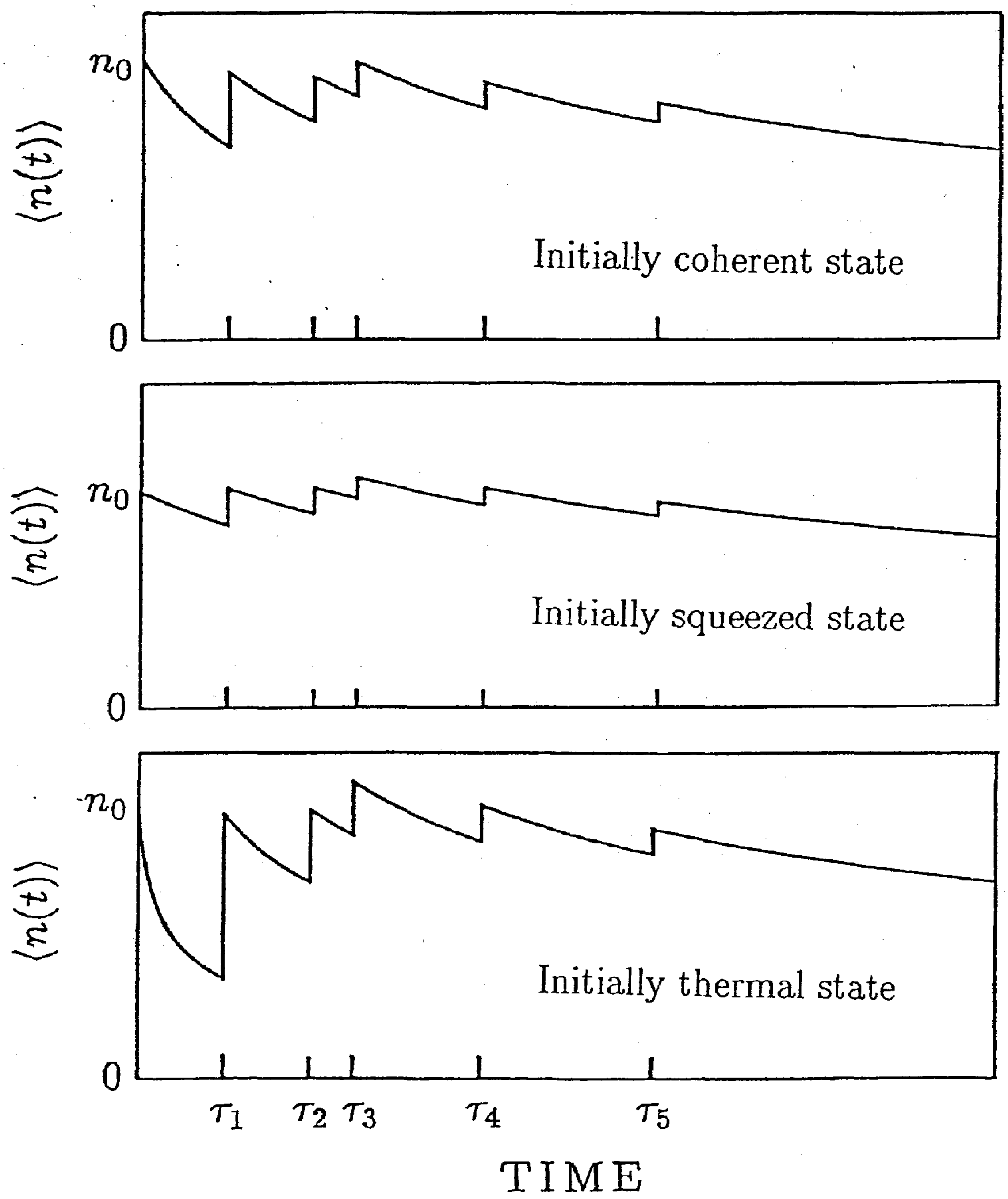


Fig.11

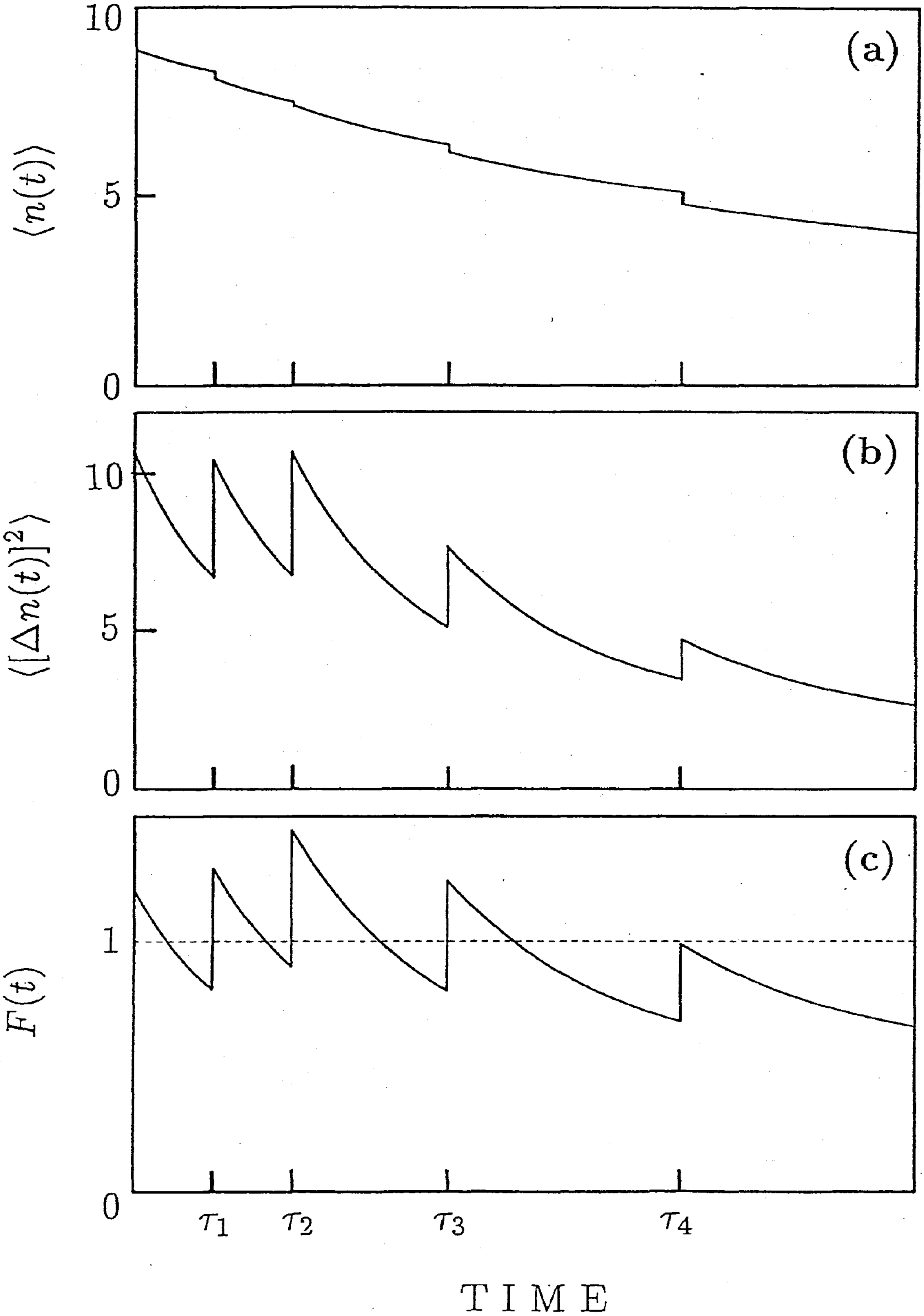

10
(a)
$\langle n(t) \rangle$
5
0
(b)
10
$\langle [\Delta n(t)]^2 \rangle$
5
0
(c)
1
$F(t)$
0
τ_1
τ_2
τ_3
τ_4
TIME

Fig.12

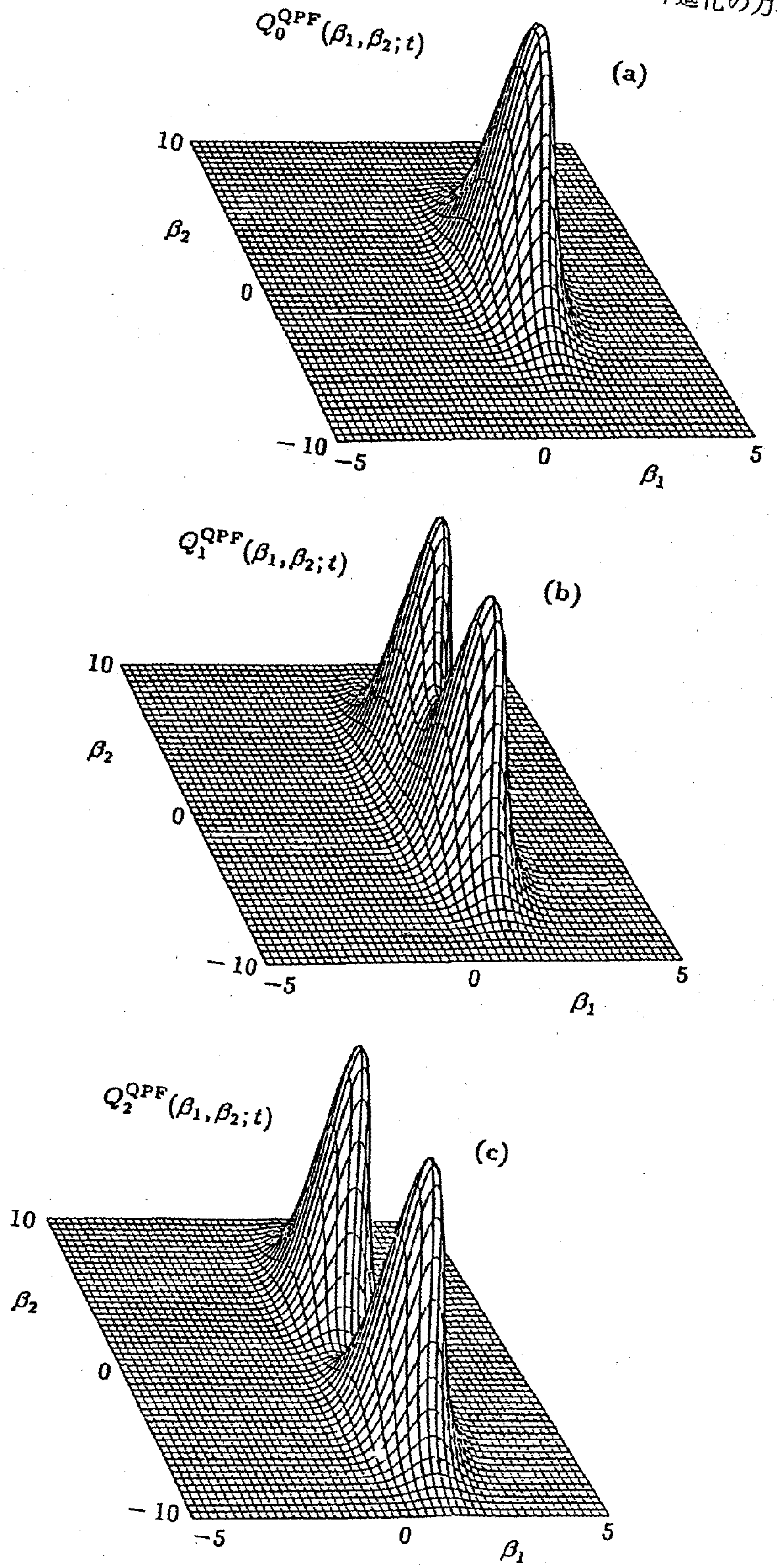

$Q_0^{\mathrm{QPF}}(\beta_1,\beta_2;t)$
(a)
10
β_2
0
−10
−5
0
5
β_1
$Q_1^{\mathrm{QPF}}(\beta_1,\beta_2;t)$
(b)
10
β_2
0
−10
−5
0
5
β_1
$Q_2^{\mathrm{QPF}}(\beta_1,\beta_2;t)$
(c)
10
β_2
0
−10
−5
0
5
β_1

Fig.13